



**MATHEMATICAL STUDIES  
STANDARD LEVEL  
PAPER 2**

Thursday 6 May 2010 (morning)

1 hour 30 minutes

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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- A graphic display calculator is required for this paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 15]

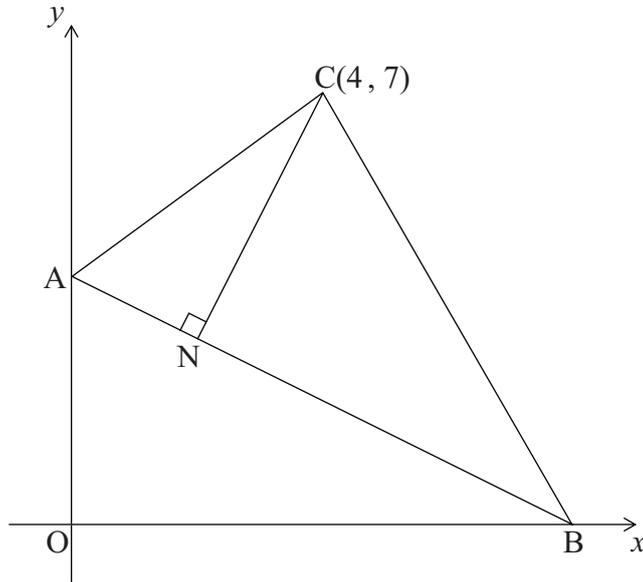
In an environmental study of plant diversity around a lake, a biologist collected data about the number of different plant species ( $y$ ) that were growing at different distances ( $x$ ) in metres from the lake shore.

Distance ( $x$ )	2	5	8	10	13	17	23	35	40
Plant species ( $y$ )	35	34	30	29	24	19	15	13	8

- (a) Draw a scatter diagram to show the data. Use a scale of 2 cm to represent 10 metres on the  $x$ -axis and 2 cm to represent 10 plant species on the  $y$ -axis. [4 marks]
- (b) Using your scatter diagram, describe the correlation between the number of different plant species and the distance from the lake shore. [1 mark]
- (c) Use your graphic display calculator to write down
- (i)  $\bar{x}$ , the mean of the distances from the lake shore;
- (ii)  $\bar{y}$ , the mean number of plant species. [2 marks]
- (d) Plot the point  $(\bar{x}, \bar{y})$  on your scatter diagram. **Label this point M.** [2 marks]
- (e) Write down the equation of the regression line  $y$  on  $x$  for the above data. [2 marks]
- (f) Draw the regression line  $y$  on  $x$  on your scatter diagram. [2 marks]
- (g) Estimate the number of plant species growing 30 metres from the lake shore. [2 marks]

2. [Maximum mark: 18]

The diagram shows triangle ABC. Point C has coordinates (4, 7) and the equation of the line AB is  $x + 2y = 8$ .

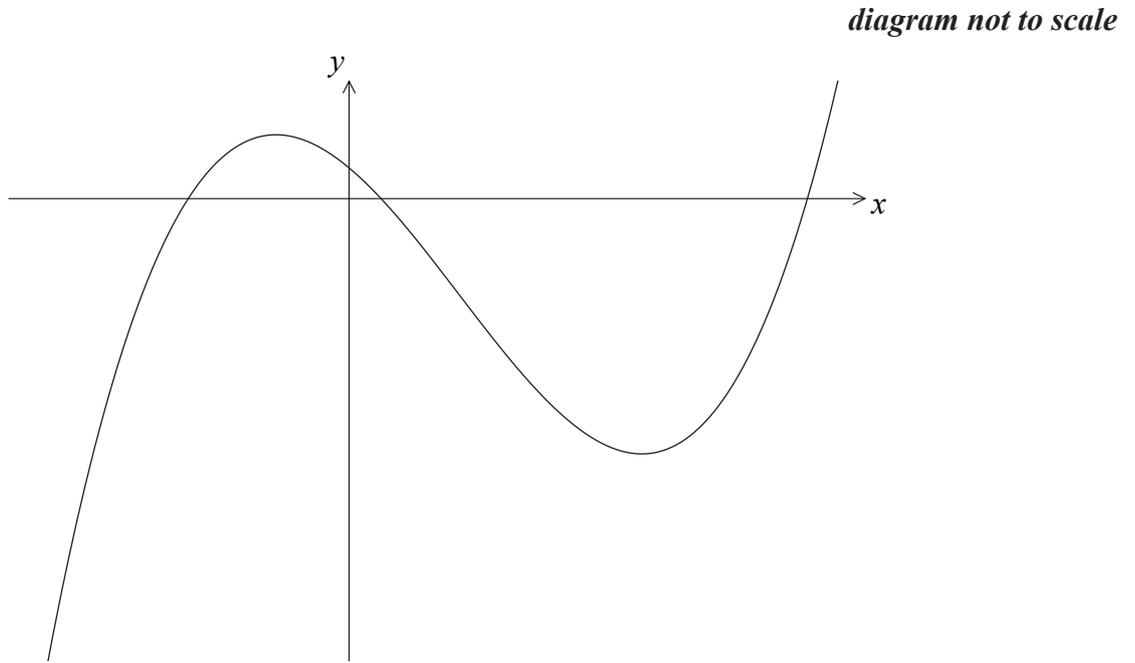


*diagram not to scale*

- (a) Find the coordinates of
- (i) A;
  - (ii) B. [2 marks]
- (b) Show that the distance between A and B is 8.94 correct to 3 significant figures. [2 marks]
- N lies on the line AB. The line CN is perpendicular to the line AB.
- (c) Find
- (i) the gradient of CN;
  - (ii) the equation of CN. [5 marks]
- (d) Calculate the coordinates of N. [3 marks]
- It is known that  $AC = 5$  and  $BC = 8.06$ .
- (e) Calculate the size of angle ACB. [3 marks]
  - (f) Calculate the area of triangle ACB. [3 marks]

3. [Maximum mark: 19]

The diagram shows a sketch of the function  $f(x) = 4x^3 - 9x^2 - 12x + 3$ .



- (a) Write down the values of  $x$  where the graph of  $f(x)$  intersects the  $x$ -axis. [3 marks]
- (b) Write down  $f'(x)$ . [3 marks]
- (c) Find the value of the local maximum of  $y = f(x)$ . [4 marks]

Let P be the point where the graph of  $f(x)$  intersects the  $y$  axis.

- (d) Write down the coordinates of P. [1 mark]
- (e) Find the gradient of the curve at P. [2 marks]

The line,  $L$ , is the tangent to the graph of  $f(x)$  at P.

- (f) Find the equation of  $L$  in the form  $y = mx + c$ . [2 marks]

There is a second point, Q, on the curve at which the tangent to  $f(x)$  is parallel to  $L$ .

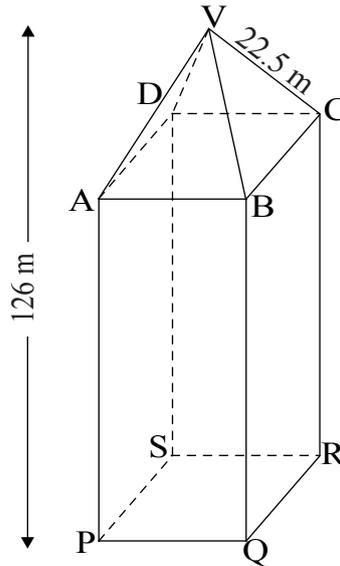
- (g) Write down the gradient of the tangent at Q. [1 mark]
- (h) Calculate the  $x$ -coordinate of Q. [3 marks]

4. [Maximum mark: 16]

The diagram shows an office tower of total height 126 metres. It consists of a square based pyramid VABCD on top of a cuboid ABCDPQRS.

V is directly above the centre of the base of the office tower.

The length of the sloping edge VC is 22.5 metres and the angle that VC makes with the base ABCD (angle VCA) is  $53.1^\circ$ .



*diagram not to scale*

- (a) (i) Write down the length of VA in metres.
- (ii) Sketch the triangle VCA showing clearly the length of VC and the size of angle VCA. [2 marks]
- (b) Show that the height of the pyramid is 18.0 metres correct to 3 significant figures. [2 marks]
- (c) Calculate the length of AC in metres. [3 marks]
- (d) Show that the length of BC is 19.1 metres correct to 3 significant figures. [2 marks]
- (e) Calculate the volume of the tower. [4 marks]

To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90 % of the volume of the tower is occupied by air and they know that  $1 \text{ m}^3$  of air weighs 1.2 kg.

- (f) Calculate the weight of air in the tower. [3 marks]

5. [Maximum mark: 22]

**Part A**

Daniel wants to invest \$25 000 for a total of three years. There are three investment options.

**Option One**                      pays simple interest at an annual rate of interest of 6 %.

**Option Two**                      pays compound interest at a nominal annual rate of interest of 5 %, compounded **annually**.

**Option Three**                    pays compound interest at a nominal annual rate of interest of 4.8 %, compounded **monthly**.

- (a) Calculate the value of his investment at the end of the third year for each investment option, **correct to two decimal places**. *[8 marks]*
- (b) Determine Daniel’s best investment option. *[1 mark]*

*(This question continues on the following page)*

(Question 5 continued)

**Part B**

An arithmetic sequence is defined as

$$u_n = 135 + 7n, \quad n = 1, 2, 3, \dots$$

(a) Calculate  $u_1$ , the first term in the sequence. [2 marks]

(b) Show that the common difference is 7. [2 marks]

$S_n$  is the sum of the first  $n$  terms of the sequence.

(c) Find an expression for  $S_n$ . Give your answer in the form  $S_n = An^2 + Bn$ , where  $A$  and  $B$  are constants. [3 marks]

The first term,  $v_1$ , of a geometric sequence is 20 and its fourth term  $v_4$  is 67.5.

(d) Show that the common ratio,  $r$ , of the geometric sequence is 1.5. [2 marks]

$T_n$  is the sum of the first  $n$  terms of the geometric sequence.

(e) Calculate  $T_7$ , the sum of the first seven terms of the geometric sequence. [2 marks]

(f) Use your graphic display calculator to find the smallest value of  $n$  for which  $T_n > S_n$ . [2 marks]